Charge pumping by electron spin resonance

2008 J. Phys.: Condens. Matter 20 055227

(http://iopscience.iop.org/0953-8984/20/5/055227)

IOP PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER
J. Phys.: Condens. Matter **20** (2008) 055227 (5pp) doi:10.1088/0953-8984/20/5/055227

Charge pumping by electron spin resonance

J Wang[1,2] and K S Chan[1]

[1] Department of Physics and Materials Science, City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong, People's Republic of China
[2] Department of Physics, Southeast University, Nanjing 210096, People's Republic of China

Received 19 November 2007, in final form 20 December 2007
Published 18 January 2008
Online at stacks.iop.org/JPhysCM/20/055227

Abstract
In the literature electron spin resonance (ESR) has been extensively studied to generate pure spin current without a charge current. In this work we address possible charge pumping by ESR in real space. Using the Keldysh formalism, we present a general charge and spin pumping result induced by rotating magnetic fields applied on a two-terminal device. It was found that when the spatial inversion symmetry of the system is broken, a nonzero charge current can flow in the system besides the spin current. At low frequency, the charge current has a quadratic frequency dependence while the spin current is linear with frequency. In the adiabatic limit, the charge current due to the breaking of spatial symmetry vanishes but the pure spin current still exists. Static disorder can greatly suppress both the pumped charge and the spin current.

Spintronics has been one of the most active research fields in recent years and aims to exploit the charge and spin degrees of freedom of electrons in spin devices, which it is hoped will be the new generation of electronic devices. The realization of quantum computation and quantum information precessing with the spin degree of freedom is another ambitious goal of spintronics [1–3]. The first challenge in this field is how to efficiently inject or generate spin current in nonmagnetic semiconductors. Although the definition of spin current is currently controversial [4, 5], especially for spin–orbit coupled systems, various schemes for generating pure spin current [6–21] have been proposed using magnetic and/or electric means, and they are believed to offer new avenues in spintronics to achieve efficient spin injection. The main methods include the spin battery based on ferromagnetic resonance [6, 7], optical injection [8, 9], magnon excitation [10, 11], equilibrium spin current [12, 13], the spin Hall effect [14, 15] etc, among which the spin pump mechanism is of great importance and has attracted a great deal of attention among researchers [16–21]. Many authors have suggested that electron spin resonance (ESR) in a quantum dot system [18–21] at zero bias can lead to a pure spin current without any charge current. The degenerate energy levels in the dot are first spin-split by a static longitudinal magnetic field, and then a transverse rotating magnetic field (RMF) is applied at the resonant frequency coupling two Zeeman split levels and making a spin flip transition between them. When the chemical potential of the lead locates between the two spin-split levels of the quantum dot, one type of spin (say the up spin) will flow spontaneously into the lower level of the QD from the lead, then the transverse rotating field can pump it into the higher level with the spin flipped, and in turn the down spin will flow out from the dot, so that a pure spin current is pumped out from the dot without external bias.

ESR can generally be used to produce a pure spin current as mentioned above; however, whether it can pump a charge current or not has not been discussed yet, even though a parametric quantum pump has been extensively discussed in the literature [22, 23]. In this paper we study charge pumping by ESR in real space by using the Keldysh formalism [24]. As is well known, the parametric quantum pump is a device that generates a flow of electrons by cyclic variation of system parameters. Therefore, for generality, we model in this work two or more out-of-phase RMFs applied to a two-terminal device with all the leads in equilibrium. At finite frequency, we present the general results for the pumped spin and charge currents in the bilinear regime. The electron and hole flow correspond, respectively, to two spin-resolved charge currents. It was found that a single RMF can give rise to not only a spin current but also a nonzero charge current when the spatial inversion symmetry of the system is broken; the charge current has a quadratic frequency dependence while the spin current is linear in frequency. We also studied the static disorder effect on the pumped current: contrary to the finding in [25] that disorder can enhance the spin current, we found that both the pumped spin and charge currents are suppressed greatly by disorder.

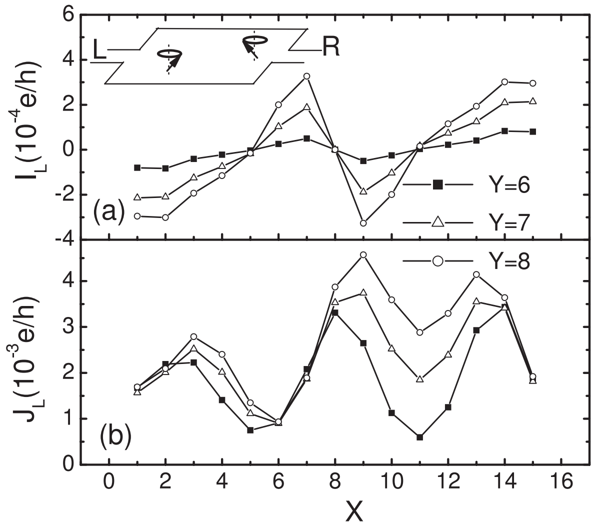

Figure 1. The pumped charge current I_L (a) and spin current J_L (b) as a function of the X coordinate of the RMF. The scattering region is a $X \times Y = 15 \times 15$ lattice, and the black rectangle, triangle, and circle lines are for $Y = 6$, 7 and 8, respectively. The inset in (a) is a two-terminal device with two RMFs applied on the scattering region. $\Delta = \omega = 0.02t$, $U_\alpha = 0.8tI$ with I the unit matrix; other parameters are in the text.

We start with a schematic two-terminal device shown in the inset of figure 1, where two ideal leads in equilibrium are connected to the scattering region of a noninteracting electron gas, the leads are spin-degenerate and the scattering region may have static magnetic or nonmagnetic disorders. Besides the longitudinal magnetic field ($B_z\hat{z}$), two transverse rotating magnetic fields $B(t) = B_{\rm rf}(\cos\omega t\hat{x} + \sin\omega t\hat{y})$ are applied on the device at different positions. These two pumping fields have the same rotation frequency but they may be out of phase. At resonance, the frequency ω is nearly equal to the Zeeman energy $\Delta = \mu_B g B_z$ by the longitudinal field with the Bohr magneton μ_B and the effective electron g factor g. The device is described by a tight binding Hamiltonian as

$$H = \sum_{\mathbf{r}\sigma}(2dt + \varepsilon_{\mathbf{r}\sigma})c^\dagger_{\mathbf{r}\sigma}c_{\mathbf{r}\sigma} - t\sum_{\mathbf{r}\mathbf{a}\sigma}(c^\dagger_{\mathbf{r}+\mathbf{a}\sigma}c_{\mathbf{r}\sigma} + \text{c.c.}) + \sum_{\sigma,i=A,B}\left(\frac{\Delta_i}{2}\sigma c^\dagger_{\mathbf{r}\sigma}c_{\mathbf{r}\sigma} + R_i c^\dagger_{\mathbf{r}\sigma}c_{\mathbf{r}\bar{\sigma}}e^{i\sigma(\omega t+\phi_i)}\right), \quad (1)$$

where $c_{\mathbf{r}\sigma}$ ($c^\dagger_{\mathbf{r}\sigma}$) is the annihilation (creation) operator of electron at position $\mathbf{r}$ with spin σ, $\sigma = \pm, \uparrow\downarrow$, $\bar{\sigma} = -\sigma$, t is the hopping energy and $\varepsilon_{\mathbf{r}\sigma}$ is the on-site energy, a is the lattice size, $i = A, B$ represent the two pumping points that may locate at any place in the scattering region, ϕ_i is the initial phase. $R_i = g\mu_B B_{\rm rf}/2$ is the ESR Rabi frequency. The first two terms in equation (1) denote the free electron Hamiltonian of the scattering region and a hard-wall potential is used in the finite size system; the last term denotes the Zeeman energy from the magnetic fields. The Hamiltonian of the lead is not shown here for it is an ideal one. The model employed here can be $d = 1$, 2 or 3 dimensional and the scattering region can be spatially regular or irregular.

Now we derive a general expression of the averaged spin resolved charge current flowing in lead $\alpha = L, R$. The spin-dependent current operator is given by $i_{\alpha\sigma} = -e\mathrm{d}N_{\alpha\sigma}/\mathrm{d}t$ where $N_{\alpha\sigma}$ is the electron number operator in lead α with spin σ. According to the Heisenberg equation, the averaged current is written as

$$I_{\alpha\sigma}(t) = \langle i_{\alpha\sigma}(t)\rangle = \frac{e}{\hbar}\,\mathrm{Tr}[U_\alpha G^<_{r\sigma,\alpha\sigma}(t,t) - G^<_{\alpha\sigma,r\sigma}(t,t)U^+_\alpha]. \quad (2)$$

Here U_α is the coupling strength between lead α and the scattering region r, which is a spin-independent matrix when the system is two- or three-dimensional. The trace is over the transverse modes. $G^<_{\alpha,r}$ is the usual lesser Green's function defined as [24]

$$G^<_{\alpha\sigma,r\sigma'}(t,t') = i\langle c^\dagger_{r\sigma'}(t')c_{\alpha\sigma}(t)\rangle, \quad (3)$$

where $\langle\cdots\rangle$ is the quantum statistical average. It is noted that the Green's function is a $2M_\alpha \times 2M_\alpha$ matrix for the spin degeneracy of 2 and M_α transverse modes in lead α. To solve the equation above, we use the perturbation theory and consider the RMF with a finite frequency ω as the perturbation. The following Dyson equation for the Keldysh formalism is used in the calculation

$$G^k(t,t') = G^{k0}(t,t') + \int \mathrm{d}t_1 G^{k0}(t,t_1)V^k(t_1)G^{k0}(t_1,t') + \cdots, \quad (4)$$

where $G^k(t,t')$ is the Green's function in Keldysh space, which is defined as [26, 27]

$$G^k(t,t') = \begin{pmatrix} G^t(t,t') & G^<(t,t') \\ G^>(t,t') & G^{\bar{t}}(t,t') \end{pmatrix}, \quad (5)$$

where

$$G^t(t,t') = -i\theta(t-t')\langle c(t)c^\dagger(t')\rangle + i\theta(t'-t)\langle c^\dagger(t')c(t)\rangle, \quad (6)$$

$$G^>(t,t') = -i\langle c(t)c^\dagger(t')\rangle, \quad (7)$$

and

$$G^{\bar{t}}(t,t') = -i\theta(t'-t)\langle c(t)c^\dagger(t')\rangle + i\theta(t-t')\langle c^\dagger(t')c(t)\rangle \quad (8)$$

are the time order, greater and anti-time order Green's functions, respectively. The perturbation potential V^k in the Dyson equation is also in the Keldysh space defined as

$$V^k(t) = \begin{pmatrix} v_i(t) & 0 \\ 0 & -v_i(t) \end{pmatrix}. \quad (9)$$

Here the time order and anti-time order components take the plus and minus perturbation potential whereas the other two components are zero because, as in most cases, the perturbation potential is instantaneous. The four component Green's functions of G^k are not fully independent and they have relations such as

$$G^t = G^< + G^r, \quad (10)$$

$$G^{\bar{t}} = G^< - G^a, \quad (11)$$

and

$$G^> = G^t - G^a. \quad (12)$$

In these expressions, $G^{\mathrm{r(a)}}$ is the usual retarded (advanced) Green's function as

$$G^{\mathrm{r(a)}}(t,t') = \mp \mathrm{i}\theta(\pm t \mp t')\langle\{c(t), c^{+}(t')\}\rangle, \tag{13}$$

and they satisfy a similar Dyson equation as G^{k} in equation (4).

Since the device is unbiased, all the electrodes have the same chemical potential and the unperturbed terms G^{k0} and $G^{\mathrm{r}0}$ and the first-order correction $\Delta^{(1)}G^{\mathrm{k(r)}}$ are not expected to give rise to a charge current. Thus we need to expand the Dyson equation to the bilinear term, which can lead to a pumped current, i.e.

$$\Delta^{(2)}G^{\mathrm{k(r)}} \sim G^{\mathrm{k(r)}0}V_i^{\mathrm{k(r)}}G^{\mathrm{k(r)}0}V_i^{\mathrm{k(r)}}G^{\mathrm{k(r)}0}. \tag{14}$$

The unperturbed Green's function $G^{\mathrm{k(r)}0}$ can be easily worked out since the system is in equilibrium, i.e. $G^{\mathrm{r}0}(E) = 1/(E - H_0 - \Sigma^r)$, where H_0 is the time-independent part of equation (1) and $\Sigma^r = \sum_\alpha U_\alpha^\dagger g_\alpha^{\mathrm{r}} U_\alpha$, with g_α^{r} being the retarded Green's function of the isolated lead α. The unperturbed lesser Green's function is given by $G^{<0} = [G^{\mathrm{a}0}(E) - G^{\mathrm{r}0}(E)]f(E)$, with $f(E)$ being the Fermi distribution function, thus G^{k0} can be calculated using equations (10)–(12). With these preparations and some algebra, we obtain the spin-resolved current in lead α as

$$\begin{aligned} I_{\alpha\sigma} = \frac{e}{\hbar}\int \frac{\mathrm{d}E}{2\pi} \sum_{i,j=A,B} \mathrm{i}\mathrm{e}^{\mathrm{i}\sigma(\phi_i-\phi_i)} R_i R_j \\ \times (f(E) - f(E+\sigma w))\,\mathrm{Tr}[\Gamma_\alpha G^{\mathrm{r}0}_{\alpha\sigma,i\sigma}(E) \\ \times (G^{\mathrm{r}0}_{i\bar{\sigma},j\bar{\sigma}}(E+\sigma w) - G^{\mathrm{a}0}_{i\bar{\sigma},j\bar{\sigma}}(E+\sigma w))G^{\mathrm{a}0}_{j\sigma,\alpha\sigma}(E)]. \end{aligned} \tag{15}$$

Here $\Gamma_\alpha = \mathrm{i}(\Sigma_\alpha^r - \Sigma_\alpha^a)$ is the line width function of lead α that denotes the electron transfer rate, and it is spin-degenerate. Equation (15) is the main result of this paper, that the rotating magnetic field can give rise to not only a steady spin current $J_\alpha = I_{\alpha\uparrow} - I_{\alpha\downarrow}$ but also a charge current $I_\alpha = I_{\alpha\uparrow} + I_{\alpha\downarrow}$. The formula derived above is similar to that for the pumped current from the usual spin-independent pumping potentials obtained by the scattering theory at finite frequency [28, 29]. For the bilinear approximation, the electron in one cycle of RMF experiences spin flip twice, so that its spin is unchanged and a pumped charge current may flow in the system; this is also the reason that the first-order correction $\Delta^{(1)}G^{\mathrm{k(r)}}$ does not contribute to the pumped current as mentioned above. The formation of the pumped current can be regarded as an excited electron–hole pair [30], due to the absorption of an energy quantum $h\omega$, flowing to different leads; thus from the equation above, the two spin-resolved charge currents correspond exactly to the electron and hole currents, respectively, and the total current is the summation of them. Using the identity

$$G^{\mathrm{a}} - G^{\mathrm{r}} = \mathrm{i}G^{\mathrm{r}}\Gamma G^{\mathrm{a}} \tag{16}$$

with $\Gamma = \sum_\alpha \Gamma_\alpha$, we can prove that the charge current is conserved:

$$\sum_{\alpha\sigma} I_{\alpha\sigma} = 0. \tag{17}$$

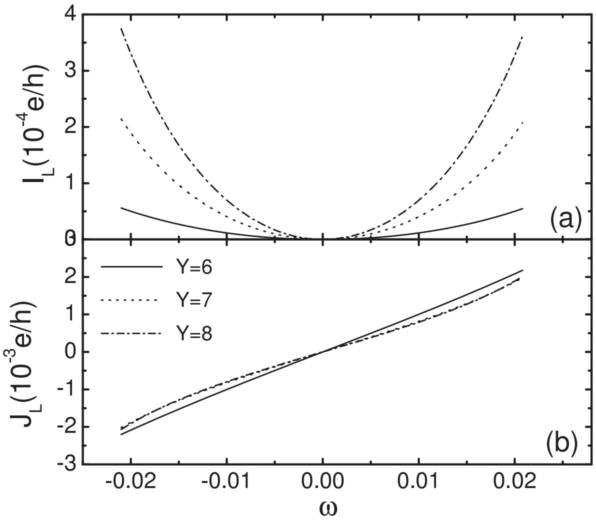

Figure 2. The frequency dependence of the pumped current I_{L} (a) and J_{L} (b). The coordinates of the RMF are taken as $X = 7$ and $Y = 6, 7, 8$. Parameters are the same as those in figure 1.

However, the spin current is not conserved, $\sum_\alpha J_\alpha \neq 0$, because the RMF can be regarded as a source of spin, in other words, the RMF can also give rise to a pure spin current in a single-terminal device.

We focus on the case of a single RMF applied on a two-terminal device. Figure 1 presents the pumped charge I_{L} and spin current J_{L} through the left lead as a function of the RMF position in the device. The scattering region is described by a 15 × 15 two-dimensional lattice. In the calculation, the hopping energy t is taken as the energy unit. The Fermi energy $E_f = 0.4t$ so that the Fermi wavevector is much larger than the lattice constant a and our model can simulate a continuum system. The Rabi frequency is not explicitly considered in the calculations since the presented results are linear with it, which can be seen from equation (15). In figure 1(a), it is shown that a nonzero pumped charge current exists in the system besides the spin current (figure 1(b)), and this charge current possesses the spatial inversion antisymmetry $I(\mathbf{r}) = -I(-\mathbf{r})$, so that it will disappear when the pumping point locates at the center of the device. In other words, if the pumping system composed of the device and the pumping point has spatial inversion symmetry (SIS) [28], no pumped charge current could form. When ESR is considered in a single-level quantum dot system which naturally has SIS, no charge current can flow [18–21]. The pumped spin current in figure 1(b) does not exhibit any symmetry because the spin current is not conserved.

Figure 2 shows I_{L} and J_{L} versus the pumping frequency ω. The charge current exhibits a quadratic frequency dependence while the spin current is linear in ω. In the low frequency limit, equation (15) could be reduced to the following expression:

$$\begin{aligned} I_{\alpha\sigma} = \frac{e\sigma\omega}{h} \sum_{i,j=A,B} R_i R_j \mathrm{i}\mathrm{e}^{\mathrm{i}\sigma(\phi_i-\phi_j)}\,\mathrm{Tr}[\Gamma_\alpha G^{\mathrm{r}0}_{\alpha\sigma,i\sigma} \\ \times (G^{\mathrm{r}0}_{i\bar{\sigma},j\bar{\sigma}} - G^{\mathrm{a}0}_{i\bar{\sigma},j\bar{\sigma}})G^{\mathrm{a}0}_{j\sigma,\alpha\sigma}]_{E=E_f} \end{aligned}$$

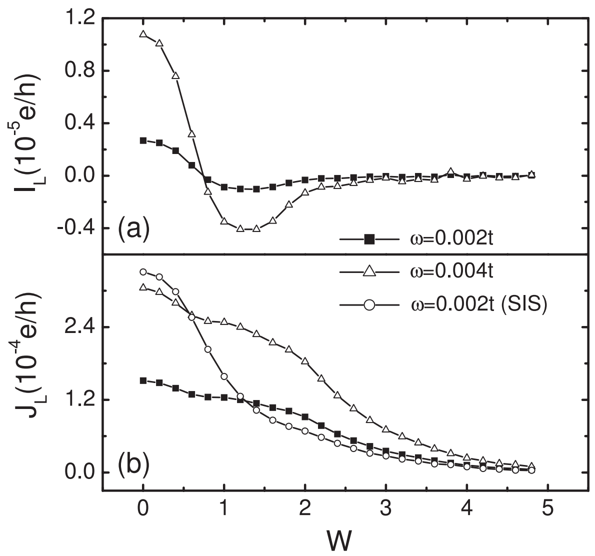

Figure 3. The pumped charge current I_L (a) and spin current J_L (b) as a function of the disorder strength W for different frequencies. In calculation, the sample average on disorder is taken over 2000 random configurations. The RMF locates at $X = 7$ and $Y = 8$ except for the 'circle' line with $X = 8$, $Y = 8$, the center of lattice. The Zeeman energy is equal to the frequency of RMF.

$$+\frac{e\omega^2}{h}\sum_{i,j=A,B} R_i R_j \mathrm{i}e^{\mathrm{i}\sigma(\phi_i-\phi_j)}\,\mathrm{Tr}\Big[\Gamma_\alpha G^{r0}_{\alpha\sigma,i\sigma} \times \frac{\partial}{\partial E}(G^{r0}_{i\bar{\sigma},j\bar{\sigma}} - G^{a0}_{i\bar{\sigma},j\bar{\sigma}})G^{a0}_{j\sigma,\alpha\sigma}\Big]_{E=E_f}. \quad (18)$$

The first term (before the plus sign) is the adiabatic result and the spin-resolved current is linear in ω, the second term is nonlinear and proportional to the square of frequency. By means of equation (16), we can show that the first (adiabatic) term does not contribute to the charge current (irrespective of the number of pumping points) but to the pure spin current, whereas the nonlinear term can result in a nonzero charge current that has a quadratic frequency dependence, hence the magnitude of the spin current is much larger than the charge current (in fact they have different units, but in this work they are simplified to have the same unit $e/\hbar$). The charge current induced by ESR is different from the usual quantum parametric pump, in which the charge current is linear in frequency in the adiabatic limit. For the case of two out-of-phase RMFs, the correlation between them can lead to $J \sim \cos\phi$ and $I \sim \sin\phi$, with ϕ being the phase difference between two RMFs; however, neither of them is larger in magnitude than a single RMF since the correlation between different pumping points is weaker than that of the same pumping point in equation (15) or (17). $J \sim \cos\phi$ indicates that a single RMF can lead to a spin current.

Hattori [25] has demonstrated that the spin current induced by RMF in the diffusive transport regime can be enhanced by disorder due to the weak localization effect. In this study, we also consider the effect of static disorder on both I and J caused by nonmagnetic impurities. In the calculation, a random on-site potential $\varepsilon_\mathbf{r}$ is introduced, which is uniformly distributed in the range $[-W/2, W/2]$ with W the disorder strength. As shown in figure 3, the computed I_L (figure 3(a)) and J_L (figure 3(b)) decreases greatly with the disorder strength W. At strong disorder, the pumped charge current can even change its direction and exhibit a fluctuation around zero, since it is related to the spatial symmetry. The decreasing trend of both pumped currents is very different from the result in [25], where the RMF was considered to apply on the whole device and there is no charge current flowing in the system. The circle line in figure 3(b) denotes the pumped pure spin current in the system that possesses SIS so that the charge current disappears. The pure spin current is shown to decrease rapidly with disorder. From the scattering perspective, the pumped current is due to the time-dependent potential, which causes inelastic scattering of electrons and subsequent nonequilibrium in-scattering and out-scattering of electrons in one lead. This can be seen from the following transformed version of equation (15):

$$I_{\alpha\sigma} = \frac{e}{h}\mathrm{Tr}\int \mathrm{d}E\sum_\beta |\Delta S_{\alpha\beta}(E+\sigma\omega)|^2(f(E) - f(E+\sigma\omega)), \quad (19)$$

where $\Delta S_{\alpha\beta}$ is the first-order correction of the scattering coefficients [29, 30] by the pumping potentials and β is the lead index. Since the scattering coefficients decrease apparently with disorder, the corresponding pumped current I and J in our scheme should be reduced rapidly as well [31]. The results found here, ESR leading to a nonzero charge current in none-SIS device, should be realizable using currently available technologies. When an RMF is applied on a device, the charge current can be measured directly by modulating the left (right) coupling strength U_α between L (R) lead and device, i.e. changing the line width function $\Gamma_{L(R)}$, which can be actually controlled by an external gate voltage [32].

In summary, we have investigated charge pumping by rotating magnetic fields in a two-terminal device. We derived a general formula of the pumped charge and spin current in real space by means of the Keldysh Green's function. A pumped current can flow in the device without external bias only when the device does not have spatial inversion symmetry, and it can disappear in the adiabatic limit. The pumped spin and charge current have, respectively, linear and quadratic frequency dependence, and both of them can be strongly reduced by disorder.

Acknowledgments

This work is supported by City University of Hong Kong Strategic Research Grant (project no. 7002029). JW is grateful for financial support from NSFC 10704016 and BK 2007564.

References

[1] Wolf S A, Awschalom D D, Buhrman R A, Daughton J M, Molnár S von, Roukes M L, Chtchelkanova A Y and Treger D M 2001 *Science* **294** 1488
[2] Awschalom D D, Loss D and Samarth N 2002 *Semiconductor Spintronics and Quantum Computation* (Berlin: Springer)
[3] Zütic I, Fabian J and Sarma S D 2004 *Rev. Mod. Phys.* **76** 323

[4] Shi J, Zhang P, Xiao D and Niu Q 2006 *Phys. Rev. Lett.* **96** 076604
[5] Wang J, Wang B, Ren W and Guo H 2006 *Phys. Rev.* B **74** 155307
[6] Brataas A, Tserkovnyak Y, Bauer G E W and Halperin B I 2002 *Phys. Rev.* B **66** 060404
[7] Wang D, Sun Q F and Guo H 2004 *Phys. Rev.* B **69** 205312
[8] Stevens M J, Smirl A L, Bhat R D R, Najmaie A, Sipe J E and van Driel H M 2003 *Phys. Rev. Lett.* **90** 136603
[9] Hubner J, Ruhle W W, Klude M, Hommel D, Bhat R D R, Sipe J E and H M van Driel 2003 *Phys. Rev. Lett.* **90** 216601
[10] Meier F and Loss D 2003 *Phys. Rev. Lett.* **90** 167204
Schütz F, Kollar M and Kopietz P 2003 *Phys. Rev. Lett.* **91** 017205
[11] Wang B, Wang J, Wang J and Xing D Y 2004 *Phys. Rev.* B **69** 174403
[12] Wang J and Chan K S 2006 *Europhys. Lett.* **75** 281
Wang J and Chan K S 2006 *Phys. Rev.* B **74** 035342
[13] Katsura H, Nagaosa N and Balatsky A V 2005 *Phys. Rev. Lett.* **95** 057205
[14] Murakami S, Nagaosa N and Zhang S-C 2003 *Science* **301** 1368
[15] Sinova J, Culcer D, Niu Q, Sinitsyn N A, Jungwirth T and MacDonald A H 2004 *Phys. Rev. Lett.* **92** 126603
[16] Sharma P and Brouwer P W 2003 *Phys. Rev. Lett.* **91** 166801
[17] Najmaie A, Sherman E Ya and Sipe J E 2005 *Phys. Rev. Lett.* **95** 056601
[18] Wang B, Wang J and Guo H 2003 *Phys. Rev.* B **67** 092408
[19] Zhang P, Xue Q-K and Xie X C 2003 *Phys. Rev. Lett.* **91** 196602
[20] Dong B, Cui H L and Lei X L 2005 *Phys. Rev. Lett.* **94** 066601
[21] Sun Q-f, Guo H and Wang J 2003 *Phys. Rev. Lett.* **90** 258301
[22] Brouwer P W 1998 *Phys. Rev.* B **58** R10135
[23] Switkes M, Marcus C M, Campman K and Gossard A G 1999 *Science* **283** 1907
[24] Keldysh L V 1965 *Sov. Phys.—JETP* **20** 1018
[25] Hattori K 2007 *Phys. Rev.* B **75** 205302
[26] Mahan G D 1993 *Many Particle Physics* (New York: Plenum)
[27] Cuevas J C, Marin-Rodero A and Levy Yeyati A 1996 *Phys. Rev.* B **54** 7366
[28] Wang B G and Wang J 2002 *Phys. Rev.* B **66** 125316
[29] Wang J, Wang B and Xing D Y 2004 *Phys. Lett.* A **326** 449
[30] Moskalets M and Buttiker M 2002 *Phys. Rev.* B **66** 035306
[31] Ermakov V, Kruchinin S, Hori H and Fujiwara A 2007 *Int. J. Mod. Phys.* B **11** 1827
[32] Long W, Sun Q F, Guo H and Wang J 2003 *Appl. Phys. Lett.* **83** 1397